**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans.z = (X-μ)/б

= (60-55)/8

= 0.625

P=0.735(from z table)

probability that the service manager cannot meet his commitment.

P=(1-0.735)

P=0.2676

Option B

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans.

1. **from** scipy **import** stats
2. **from** scipy.stats **import** norm
3. In [10]:
4. *# A. More employees at the processing center are older than 44 than between 38 and 44.*
5. In [8]:
6. *# p(X>44); Employees older than 44 yrs of age*
7. 1**-**stats**.**norm**.**cdf(44,loc**=**38,scale**=**6)
8. Out[8]:
9. 0.15865525393145707
10. In [9]:
11. *# p(38<X<44); Employees between 38 to 44 yrs of age*
12. stats**.**norm**.**cdf(44,38,6)**-**stats**.**norm**.**cdf(38,38,6)
13. Out[9]:
14. 0.3413447460685429
15. In [11]:
16. *# B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.*
17. In [12]:
18. *# P(X<30); Employees under 30 yrs of age*
19. stats**.**norm**.**cdf(30,38,6)
20. Out[12]:
21. 0.09121121972586788
22. In [13]:
23. *# No. of employees attending training program from 400 nos. is N\*P(X<30)*
24. 400**\***stats**.**norm**.**cdf(30,38,6)
25. Out[13]:
26. 36.484487890347154
27. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.
    * multiplication, we get
    * 2x1=N(2µ,22σ2)2X1=N(2σ,4σ2)
    * addition, we get
    * X1+X2=N(µ+µ,σ2+σ2)=N(2µ,2σ2)
    * And the difference between the two is given by
    * 2X1-(X1+X2)=N(2µ-2µ,2σ21+4σ22)=N(0,6σ2)
    * The mean of  2X1 and  X1+X2 is same but the var(σ2) of 2X1  is 2 times more than the variance of  X1+X2.

4.Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

1. 90.5, 105.9
2. 80.2, 119.8
3. 22, 78
4. 48.5, 151.5
5. 90.1, 109.9

Ans. **from** scipy **import** stats

**from** scipy.stats **import** norm

In [3]:

stats**.**norm**.**interval(0.99,100,20),

Out[3]:

(48.48341392902199, 151.516586070978)

option D

5.Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45

1. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
2. Specify the 5th percentile of profit (in Rupees) for the company
3. Which of the two divisions has a larger probability of making a loss in a given year?

Ans. **import** numpy **as** np

**from** scipy **import** stats

**from** scipy.stats **import** norm

In [2]:

*# Mean profits from two different divisions of a company = Mean1 + Mean2*

Mean **=** 5**+**7

print('Mean Profit is Rs', Mean**\***45,'Million')

Mean Profit is Rs 540 Million

In [3]:

*# Variance of profits from two different divisions of a company = SD^2 = SD1^2 + SD2^2*

SD **=** np**.**sqrt((9)**+**(16))

print('Standard Deviation is Rs', SD**\***45, 'Million')

Standard Deviation is Rs 225.0 Million

**In [4]:**

***# A. Specify a Rupee range (centered on the mean) such that it contains*** *95% probability for the annual profit of the company.*

print('Range is Rs',(stats**.**norm**.**interval(0.95,540,225)),'in Millions')

Range is Rs (99.00810347848784, 980.9918965215122) in Millions

In [7]:

***# B. Specify the 5th percentile of profit (in Rupees) for the company***

*# To compute 5th Percentile, we use the formula X=μ + Zσ; wherein from z table, 5 percentile = -1.645*

X**=** 540**+**(**-**1.645)**\***(225)

print('5th percentile of profit (in Million Rupees) is',np**.**round(X,))

5th percentile of profit (in Million Rupees) is 170.0

**In [6]:**

***# C. Which of the two divisions has a larger probability of making a loss in a given year?***

In [7]:

*# Probability of Division 1 making a loss P(X<0)*

stats**.**norm**.**cdf(0,5,3)

Out[7]:

0.0477903522728147

In [8]:

*# Probability of Division 2 making a loss P(X<0)*

stats**.**norm**.**cdf(0,7,4)

Out[8]:

0.040059156863817086